Computer Modeling of Transmission Line Networks

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Abstract

A technique for modeling complex transmission line networks is presented. A general technique is introduced for solving complex transmission line systems. A tree data structure and algorithm for representing, and simultaneously solving for all nodes within the transmission line network is developed. The method is based on representing the network as a recursive tree structure and solving for the voltage, current, and impedance at each node using recursive programming techniques. First, all frequency dependent parameters within the tree structure are updated, then in a post-order traversing of the tree, the impedances at each node are computed followed by a pre-order traversing of the tree to compute node voltages and currents. A tapered transmission line model was used to verify the algorithm. This paper is based work done at the Center for Communication and Signal Processing (CCSP) at North Carolina State University by the author in 1987. The work is updated to show that the computer model and algorithm accurately models a 10 GHz microstrip band pass filter. The project is hosted on GitHub: https://github.com/silicondsp/TransNetCalc

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Chapter 1

Introduction

A technique for modeling complex transmission line networks is presented. A general technique is introduced for solving complex transmission line systems. Computer programs for modeling transmission line networks have been written using ABCD parameters [8]. In this paper a technique in which the frequency response is simultaneously obtained at all nodes within the network is presented. This paper is based on work done at the Center for Communication and Signal Processing (CCSP) at North Carolina State University by the author in 1987. A major objective of this paper is to bring all the work into single comprehensive report. In addition, a lot of the original work was wriiten in Word (1987) and has since become unreadable. So all the work has been converted to Latex including equations, and many of the original figures have been resurrected. For the Latex file visit https://aj7bf.com blog.

Chapter 2

Transmission Line Networks

Consider the basic problem of simulating pulse transmission through a loaded transmission line. Assume that the pulse of interest is bandlimited with a cutoff frequency of f_c . We can obtain the pulse response by first computing the frequency response of the network at equal intervals, then we perform a complex multiplication of the frequency response of the pulse and the transmission line network as calculated, and finally the inverse FFT of the result yields the time domain pulse response. Actually, the impulse response can also be obtained by computing the inverse FFT of the frequency response. Therefore, as a first step in calculating the frequency response of the network, we analyze the network response to a single sinusoid of frequency f_0. Consider the loaded transmission line connected to the generator E_g through a source impedance Z_s as shown in Figure 1 [5].

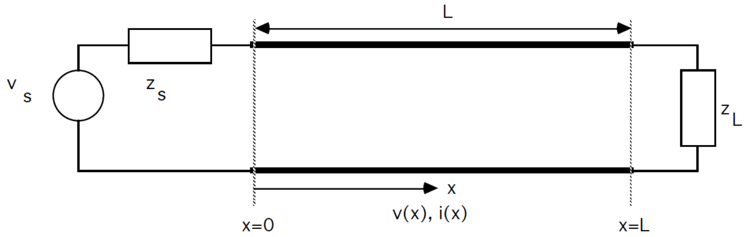


Figure 1: Generator connected to loaded transmission line

The voltage and current at any point on the transmission line can be obtained from the following expressions:

$$v(x) = \frac{v_s Z_0}{Z_0 + Z_s} e^{-\gamma x} \frac{1 + \Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_s \Gamma_L e^{-2\gamma L}} \tag{1}$$

$$i(x) = \frac{v_s}{Z_0 + Z_s} e^{-\gamma x} \frac{1 - \Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_s \Gamma_L e^{-2\gamma L}} \tag{2}$$

In the above expressions

$$\gamma = \sqrt{(r + j\omega l)(g + j\omega c)} \tag{3}$$

is the propagation constant and

$$Z_0 = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \tag{4}$$

is the characteristic impedance of the transmission line. The expressions for the source and load reflection coefficients are,

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \tag{5}$$

$$\Gamma_S = \frac{Z_s - Z_0}{Z_s + Z_0} \tag{6}$$

The expression for $v(x)$ includes the superposition of all waves reflecting from the source and load mismatches. This can be seen by a Taylor series expansion of (1)

$$v(x) = \frac{v_s Z_0}{Z_s + Z_0}[e^{-\gamma x} + \Gamma_L e^{-\gamma(L-x)} + \Gamma_L \Gamma_s e^{-\gamma(2L+x)} + \Gamma_L^2 \Gamma_s e^{-\gamma(3L-x)} + \Gamma_L^2 \Gamma_s^2 e^{-\gamma(3L+x)} + ...] \tag{7}$$

To obtain the shape of the pulse at the load we evaluate $v(L)$ at frequencies from $f = 0$ to $f = f_c$ in discrete steps where f_c is the cutoff frequency of the bandlimited pulse. The number of points must be a power of 2 such that the inverse FFT may be used to obtain the sampled pulse response at the load.

Consider now the case where the boundary voltage and current are known on a section of transmission line. See Figure 2 . Evaluate $v(0)$ in (1) and then compute.

$$\frac{v(x)}{v(0)} = e^{-\gamma x} \frac{1 + \Gamma_L e^{-2\gamma(L-x)}}{1 + \Gamma_L e^{-2\gamma L}} \tag{8}$$

Also

$$\frac{i(x)}{i(0)} = e^{-\gamma x} \frac{1 - \Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_L e^{-2\gamma L}} \tag{9}$$

Thus, using (8) and (9) the voltage and current can be evaluated at any point on the transmission line given the boundary voltage and current.

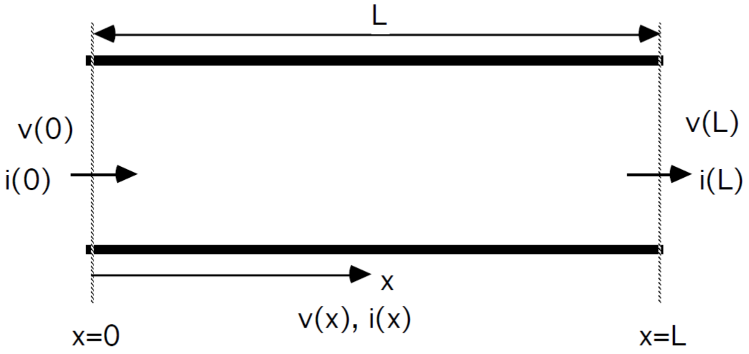


Figure 2: Section of transmission line with boundary voltages and currents

With the above preliminaries, we will examine the simple network in Figure 3 and present a methodology for its solution. In Figure 3, the nodes have been labeled n1 through n5. To solve this network, that is to obtain the voltage and current at each node and at any location within the network, consider equation (1). This equation suggests that if the impedance at node n1 was known then the voltage and current at node n1 can be calculated from the generator and source impedance. Thus, the first step is to obtain the impedance at n1. This impedance is seen to consist of the parallel combination of the impedance looking into n5 and n2 from n1.

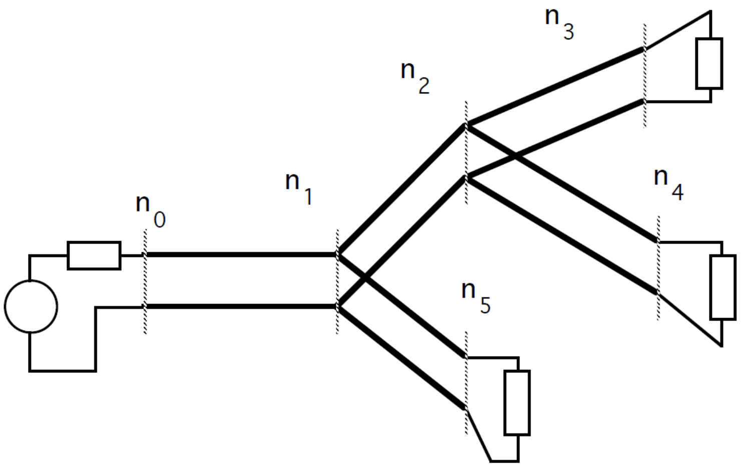


Figure 3: Example transmission line network

These impedances can be obtained by noting that (Figure 4),

$$Z_{in}(x) = \frac{1 - +\Gamma_L e^{-2\gamma(L-x)}}{1 - \Gamma_L e^{-2\gamma(L-x)}} Z_0 \tag{10}$$

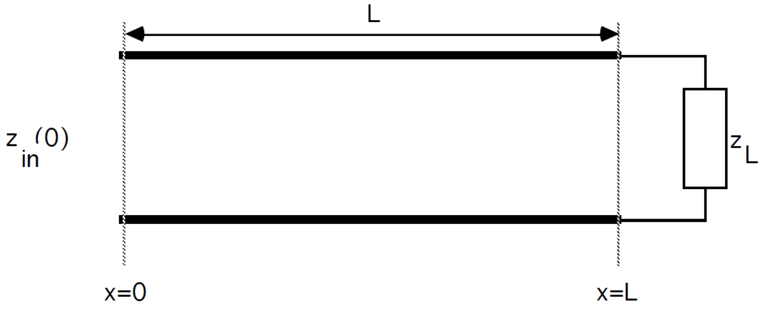


Figure 4: Input impedance of a loaded transmission line

Thus, the first step is to calculate the impedances looking into n3 and n4 from n2. The parallel combination forms the impedance at n2. The impedance at n1 is thus calculated by the parallel combination of the impedances looking into n2 and n5.

Therefore, the following methodology is suggested for solving the network. In the first pass, starting from the three loaded end nodes, the impedances are calculated and the parallel combination of these impedances at the parent node forms the parent node impedance. Working backward in this manner, the impedance at the root node (n1 in the example) is calculated. Using 1 the voltage and current at the root node n1 is calculated. Using (8) and (9) and the boundary voltages and currents, calculated at the parent node, the voltage and current at each node in the network can be calculated. Note that the current at each node is split into two currents flowing into each node.

In the case of propagation through layers of different medium, there are no branches. This situation is highlighted in Figure 5. In this case the impedance at node n5 is computed first. Next, the impedance at node n4 is computed and so forth until the total impedance looking into the network is obtained. In the next phase, the voltage and currents are computed starting from the source, node n1 and moving towards the load.

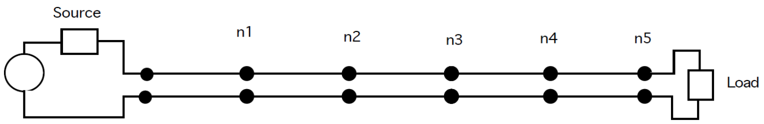


Figure 5: Network of cascaded sections

Chapter 3

Recursive Programming and Data Structures

To introduce the algorithm for solving a complex transmission line network, we first consider the case where the network is limited to the binary tree structure shown in Figure 6 . In the figure, the generator is connected to the root of the tree through a source impedance Z_s. The tree consists of nodes which are either parents or leaves. A leaf is a node which is terminated on a load. For example, n3, n4, n6, n7, n9, n11, and n12. Parent nodes have two branches. A left branch and a right branch. Nodes n1,n2, n5, n8, and n10 are parent nodes.

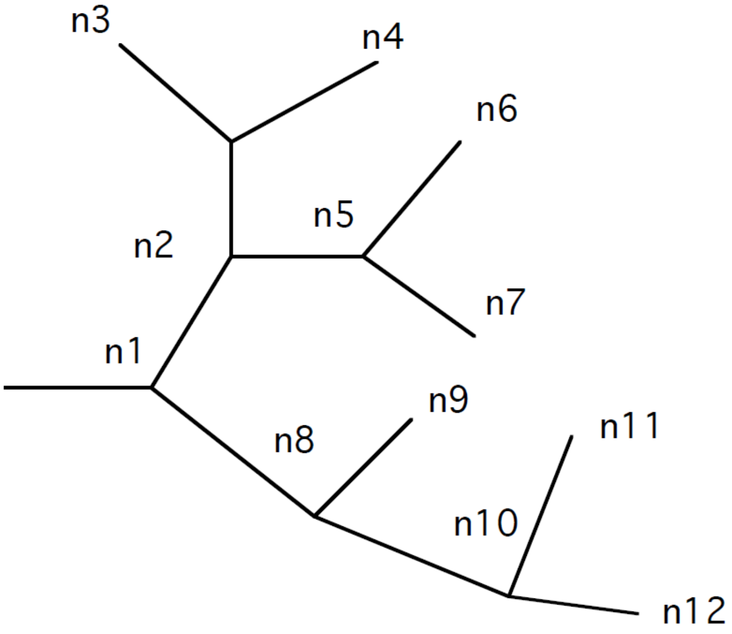


Figure 6: Transmission line network as a tree structure

3.1 Node Data Structure

In general each branch represents a transmission line with different characteristics and lengths. Each section of transmission line is associated with the node on which it terminates. Thus, the section of transmission line from the generator to the root node n1 is described in the data structure pointed to by n1. This concept is described below. Each node has an associated data structure which occupies memory locations. A pointer can be defined which points to the data structure in memory. As nodes are added to the tree, memory is dynamically allocated for the data structure and a pointer is defined.

Within the data structure definition are three pointers to data structures of the same type. Thus the data structure is recursive. Two pointers point to the left and right nodes while the third pointer points to the parent node. Three cases are immediately evident. If the node is a leaf then the left and right node pointers are NULL. Otherwise, they will point to the left and right child nodes attached to the node. If the node is the root node, then the pointer to the parent will be NULL.

The other data types within the structure represent data necessary to describe the node. These can be classified into two groups. One group defines the name of the node and the characteristics of the transmission line (e.g., r,l,c,g and Z0 and g). The other group represents data which are calculated and depend on the network. These include the voltage at the node, the current flowing into the right and left nodes, and the impedances looking into the nodes.

```
struct node {
struct node *left;
struct node *right;
struct node *parent;
char name [16];
float r,l,c,g;
float length
complex Z_Left;
complex Z_Right;
complex ZL;
complex node_voltage;
complex left_current;
complex right_current;
complex input_current;
complex Z0;
complex gamma;.
}
```

3.2 Traversing Trees

There are general methods for traversing trees [4]. We will apply two of these methods to solve the tree network.

Postorder Listing

Postorder traversing of trees is illustrated in Figure 7. This method is useful in the first pass needed prior to solving for the voltages and currents of the network. As pointed out earlier the impedances at each node must be computed. Thus in Figure 7, the impedance at 3 is the parallel combination of the impedances of the loaded transmission lines 1 and 2. Similarly, the impedance at 6 is computed from 4 and 5. Once the impedance of 3 and 6 are computed, the impedance at 7 can be calculated and so on. Careful study of the figure will show that the numbering schemes corresponds to the order in which the impedance calculations must be carried out. This order of traversing the tree is termed postorder listing. The method is summarized below [9]:

(1) If a tree is composed of only a single node, the post order listing consists of just that single node.

(2) If a tree consists of more than one node, the postorder listing consists of the postorder listing of each subtree, in left-to-right order, followed by the root.

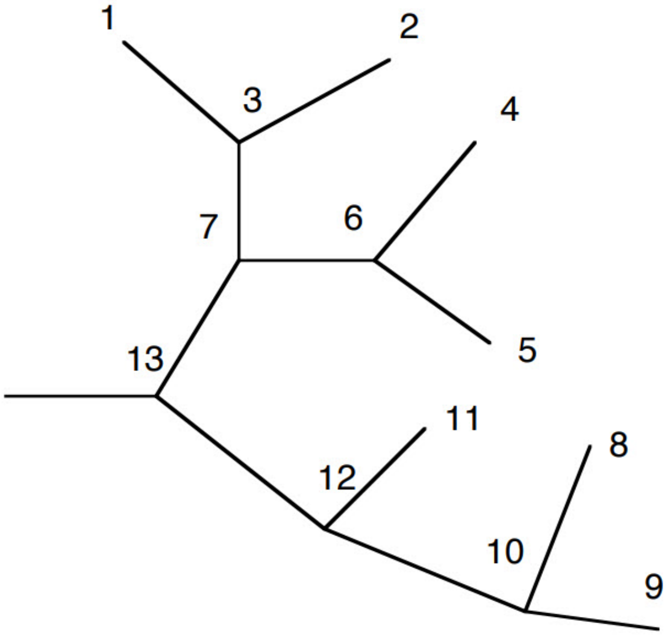


Figure 7: Post-order traversing of tree for impedance calculations

Preorder Listing

This method is used in the calculation of the voltages and currents at each node once the impedances have been determined. Preorder listing is illustrated in Figure 8. Thus, once the boundary voltage at node 1 is known, the voltage at node 2 can be computed (since the impedance at 2 is also known from the first postorder traversing in computing the impedance). From 2, the voltage at 3 and 6 can be computed and so on. The preorder listing method is summarized below [9]:

(1) If a tree is composed of a single node, the preorder listing consists of just that single node.

(2) If a tree consists of more than one node, the preorder listing consists of the root, followed by the preorder listing of each sub tree in left-to-right order.

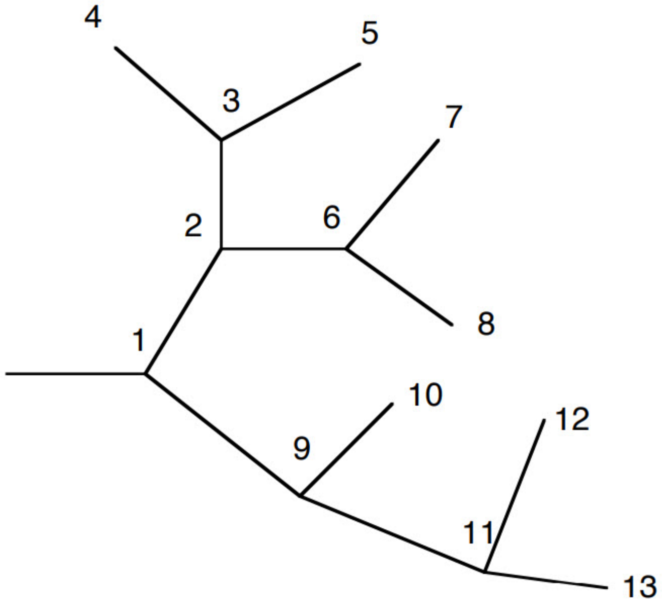


Figure 8: Pre-order traversing of tree for current and voltage calculations

For C code algorithms for reperenting the tree structure and post-order and pre-order traversing of the tree see [9].

A detailed description of the algorithm to solve the binary tree representation of transmission line networks is available in [1] including using a TCL script interpreter. The original work is documented in [5].

Chapter 4

Accuracy of Modeling Continuous Taper with Cascaded Sections.

The purpose of this modeling and simulation is to determine whether the results produced by the Transmission Line Networking Algorithm and Program$(TransNetCalc)$ for a transmission line with a tapered characteristic impedance converges to the true solution as the number of sections is increased. In addition, it is desired to determine the accuracy to be expected in a given simulation as a function of the number of sections used in $TransNetCalc$. Consider the transmission line in Figure 9 with an exponentially tapered characteristic impedance.

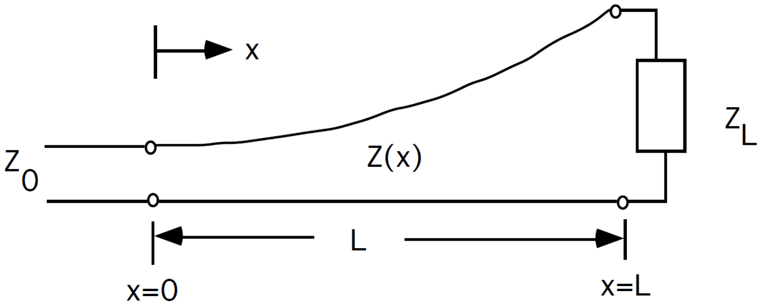


Figure 9: Exponentially tapered characteristic impedance transmission line

In our simulation we used a source impedance of 100Ω and a load impedance of 500Ω. Also, the total length L=10m. We used sections of two wire cable with geometry on Figure 10.

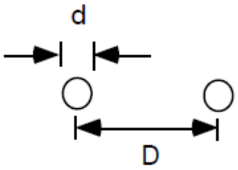


Figure 10: Two conductor parallel transmission line

The characteristic impedance is,

$$Z_0 = 120 ln(\frac{2D}{d}) \tag{11}$$

For the continuous exponential taper

$$ln(Z) = \frac{x}{L} = ln(Z_L) \tag{12}$$

(We note that using the symbol “L” for Load, Inductance, and Length, will not be an issue hoping the context will provide identity). The exact differential equation relating the reflection coefficient to location along the line is (Riccati equation, see [7])

$$\frac{d\Gamma}{dx} = j2\beta\Gamma - \frac{1}{2}(1 - \Gamma^2)\frac{d[ln(Z_L)]}{dx} \tag{13}$$

The exact solution to this differential equation for the input reflection coefficient for the case of an exponential taper is

$$\Gamma_i = \frac{A\sin(\frac{BL}{2})}{B\cos(\frac{BL}{2}) + j2\beta\sin(\frac{BL}{2})} \tag{14}$$

$$A = \frac{ln(Z_L)}{L}, B = \sqrt{4\beta^2 - A^2} \tag{15}$$

In order to test the program $TransNetCalc$[2], we cascaded sections of two wire transmission lines. The program $TransNetCalc$[2] requires values of inductance L and capacitance C.

$$L = \frac{Z_0}{v_c} Z_L^{\frac{x}{L}} \tag{16}$$

$$C = \frac{1}{v_c Z_0} Z_L^{\frac{-x}{L}} \tag{17}$$

where v_c is the velocity of light in free space. In Figure 11 a comparison of the results from $TransNetCalc$[2] and the exact solution to the Riccati equation is presented. The best comparison is in the Figure 12 which is a “zoomed in” view of Figure 11 . A simulation was performed with 500 sections and the result could not be distinguished from the exact solution. The conclusion is that $TransNetCalc$[2] produces the exact solution asymptotically as the number of sections is increased. The reason for this asymptotic exactness is that $TransNetCalc$[2] includes the effects of all reflections, not a simple approximation where second and higher order reflections are neglected.

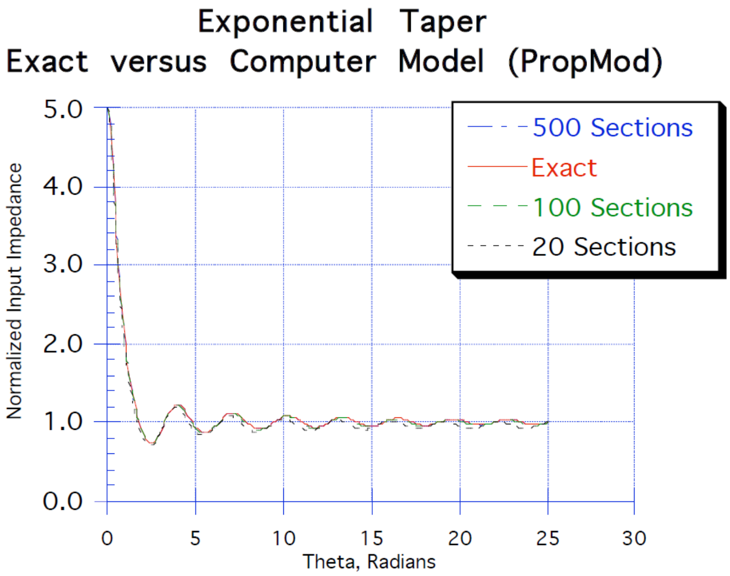


Figure 11: Comparison of $TransNetCalc$[2] Computer Model and Exact Result

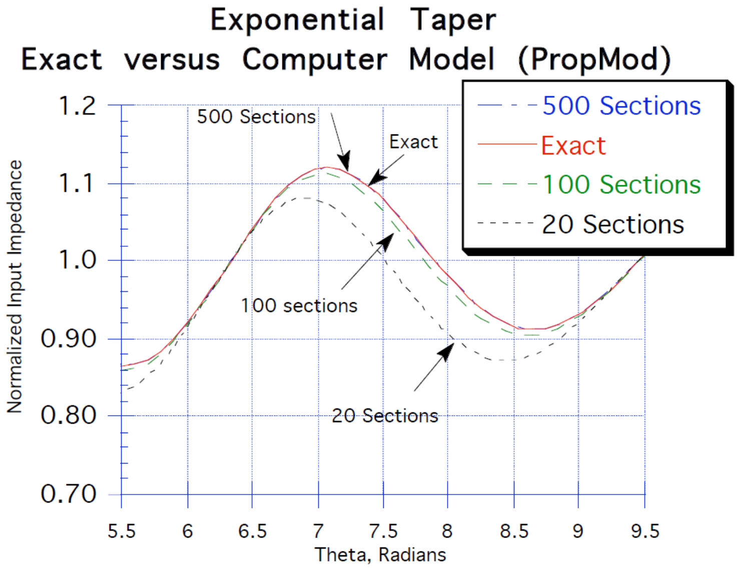


Figure 12: Zoomed In Comparison of $TransNetCalc$[2] Computer Model and Exact Result

Chapter 5

Modeling A Double Stub Tuner

The following double stub tuner is described in [6].

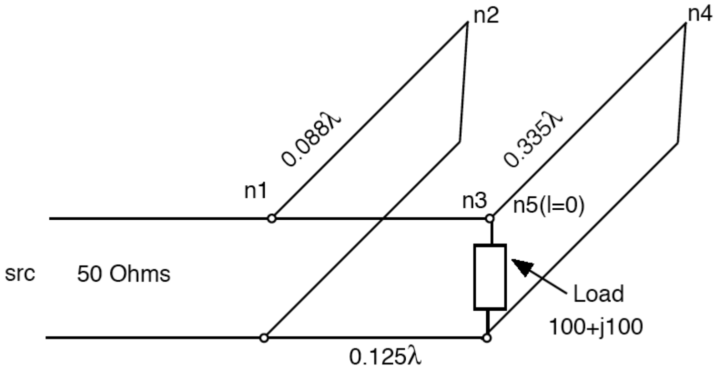


Figure 13: Double Stub Tuner

The double stub tuner matches the load to 50 Ohms. In modeling and simulating this network, we will use the coaxial transmission line. The transmission line parameters are for a 50 Ohm characteristic impedance. The network is matched at 10MHz. The wavelength is 20m.

The network is represented by the tree structure in the following ASCII file ($TransNetCalc$[2]).

```
n1 n2 n3
n3 n4 n5
n2
n4
n5
end
n1 coax1 30
n2 coax1 1.76 0 0
n3 coax1 2.5
n4 coax1 6.6 0 0
n5 R100_L0.0000015915 0 open
```

Associated with transmission line networks is an ASCII file with transmission line types and parameters.

```
#19m 0.053437723 5.530183e-7 5.157361e-11 1.031472e-9
#22m 0.10749682  5.405909e-7 5.157361e-11 1.031472e-9
#24m 0.17025507  5.903004e-7 5.157361e-11 1.031472e-9
#26m 0.27340231  6.213688e-7 5.157361e-11 1.031472e-9
coax1         0.001         0.0035      2.25      0.001
balsh1   0.001  0.0035 2.25 0.002
wireabg1 0.001 0.1 2.25 0.0
paral1   0.001 0.01 2.25 0.0
```

In the case of the double stub tuner the coaxial line is described by the parameters shown in Figure 14.

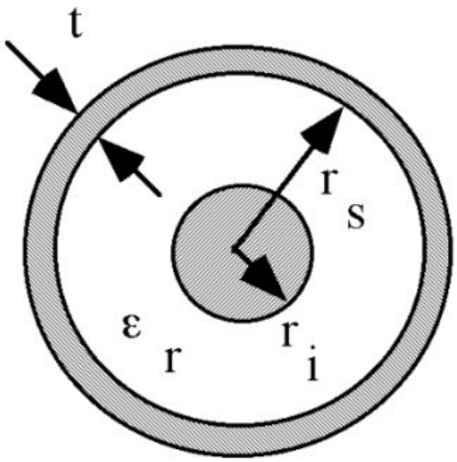


Figure 14: Coax

The performance of the double stub tuner is shown in the Figure 15. The VSWR is plotted against the frequency (+- 10% of 10MHz).

This result was obtained using $TransNetCalc$ with integrated TCL interpreter which makes all calculations of the network including setting frequency and node parameters available through TCL commands.

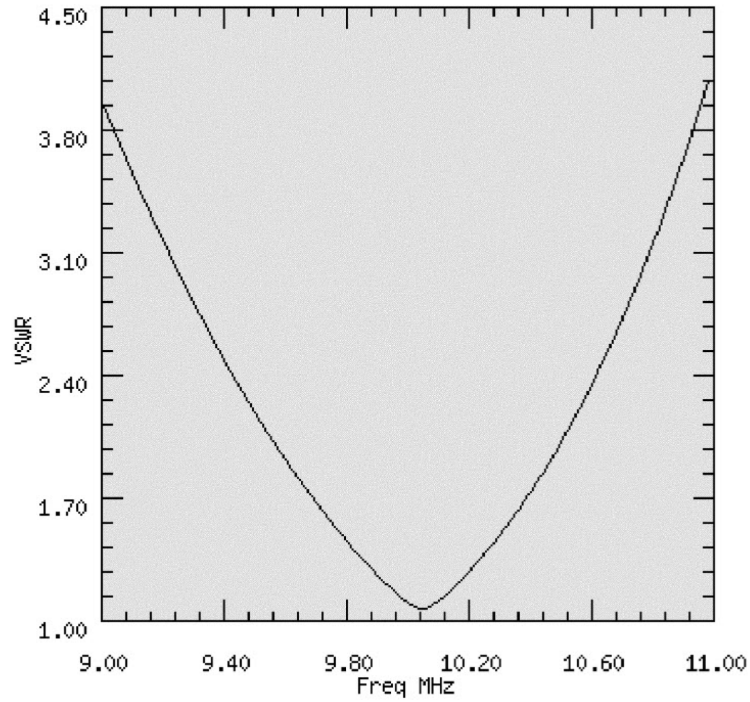


Figure 15: VSWR versus Frequency

The performance of the double stub tuner as a function of the variation of the stub length for node n2 is shown in the Figure 16. The result is calculated using $TransNetCalc$ and integrated TCL. The length is varied between +-10% of the tuned length at 10MHz.

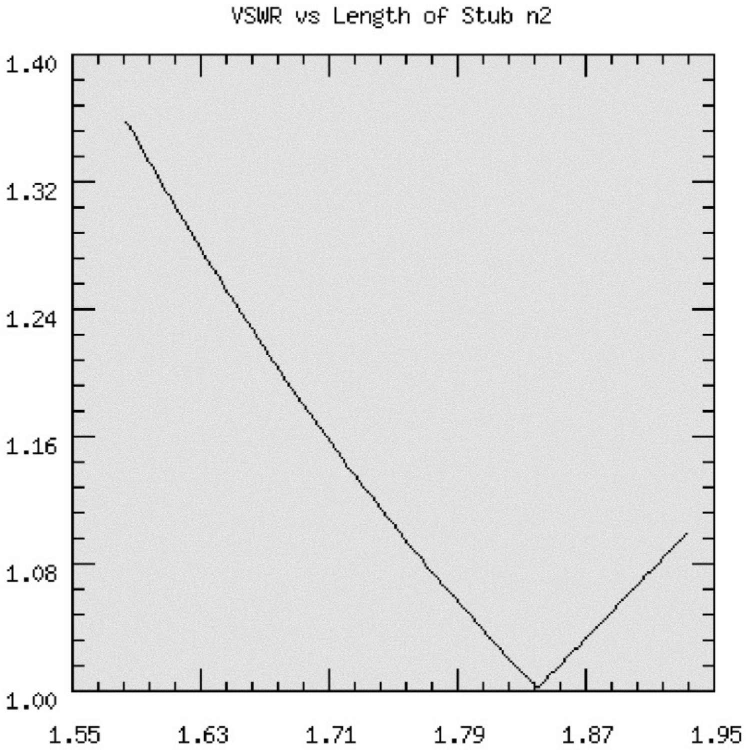


Figure 16: VSWR versus Length

Chapter 6

Modeling X Band Microstrip Filters

6.1 XBand Bandpass Filter 10 GHz

The design of a 10GHz Bandpass Filter will be presented. This filter plays a critical role in the design of XBand Communication Systems [3]. Figure 17 shows the Band Pass Filter centered at 10 GHz. The frequency response of the filter is shown in Figure 18.

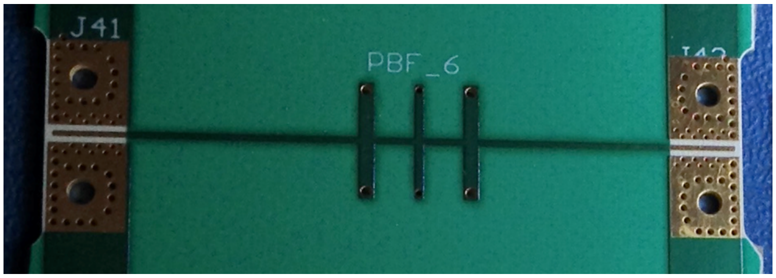


Figure 17: Microstrip 10 GHz Bandpass Filter Coupon Rogers 4530. Note the Shorts at the End of the Stubs

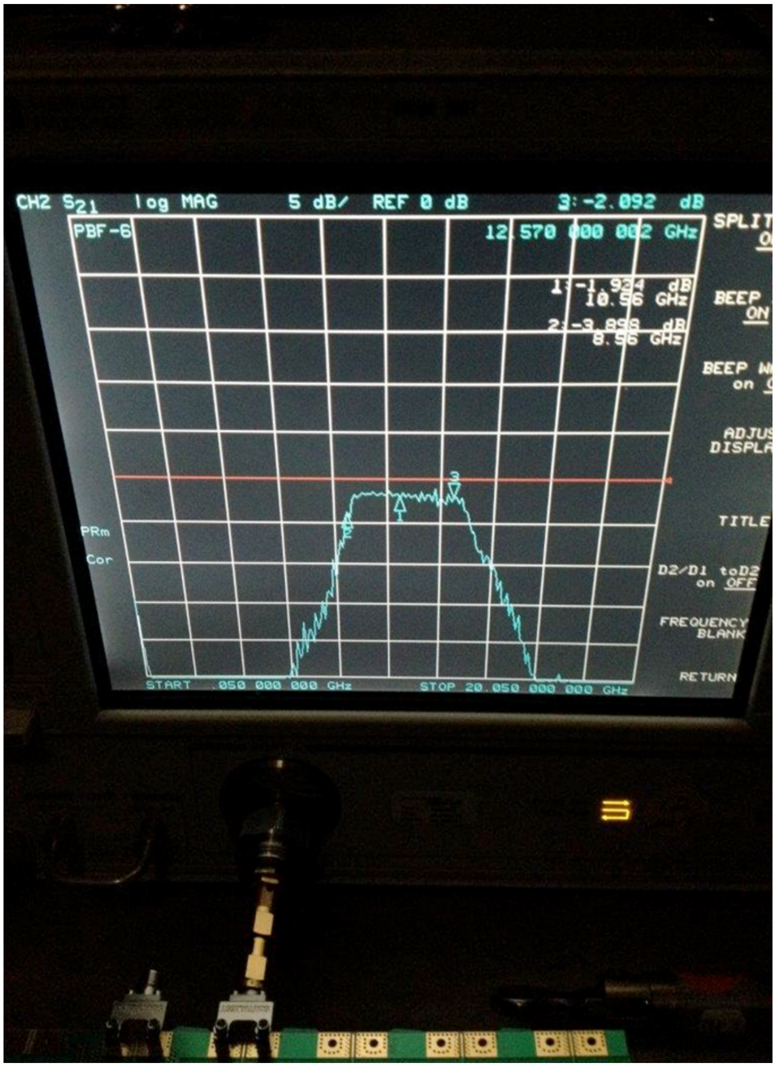
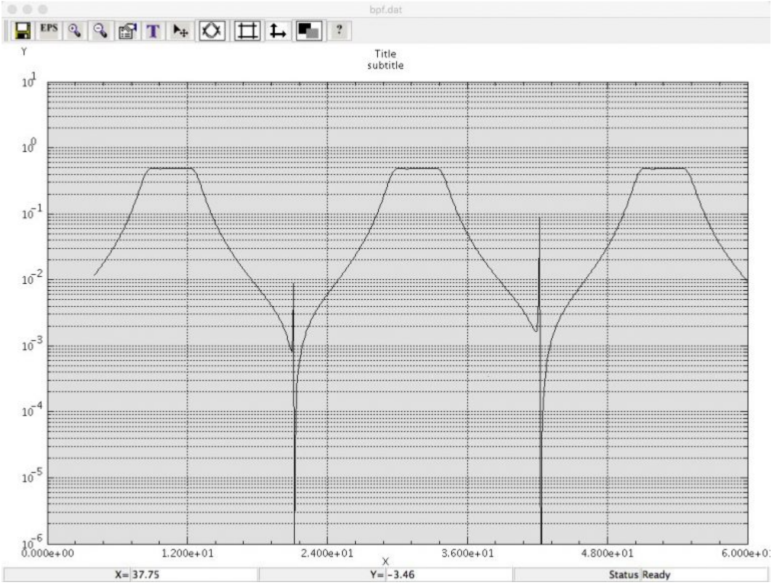


Figure 18: Network Analyzer 20 GHz Span Microstrip 10 GHz Bandpass Filter Coupon Rogers 4530

Before tapeout of the Band Pass Filter, its frequency response was verified using *TransNetCalc* [2]. The results are shown in Figure 19. The computer modeling and hardware measurement match.



X= 37.75

Y= -3.46

Status Ready

Figure 19: Calculated Frequency Response Using TransNetCalc Which Matches the Measured Frequency Repsonse in Figure 18

The PCB stackup is shown in Figure 20.

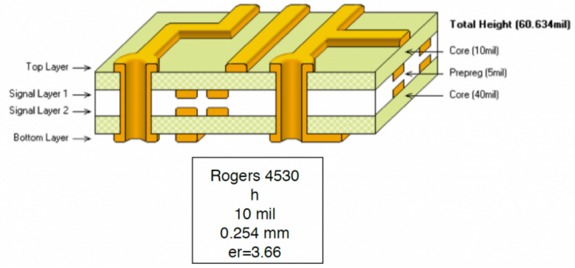


Figure 20: Stack Up Bandpass Filter Coupon Rogers 4530

The complete details of the microstrip Band Pass Filter are shown in Figure 21.

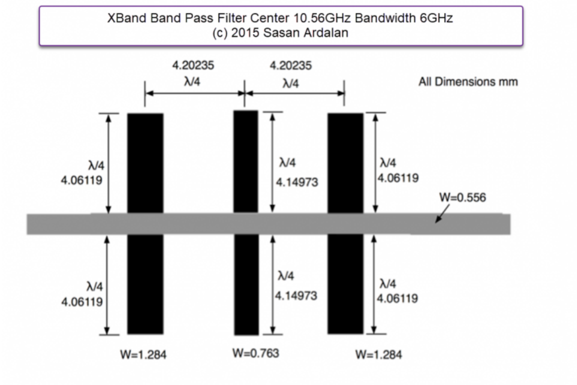


Figure 21: Microstrip 10 GHz Bandpass Filter

The detailed description of the $TransNetCalc$ modeling and TCL scripting of the Band Pass Filter is provided in Appendix A.

For the design and modeling of a 8.34GHz Band Stop Filter and more information on the XBand Communications System [3] visit https://www.aj7bf.com.

Chapter 7

Conclusions

In this paper, a computer program is described which simultaneously solves for all nodes within complex networks of transmission lines. A tree data structure was introduced for representing the network in the computer. Recursive procedures were presented for traversing the tree data structure to compute the impedance, voltage and current at each node within the network. The application of the computer modeling program ($TransNetCalc$ [2]) to a double stub tuner and a 10GHz microstrip Band Pass Filter was presented. It is shown that the $TransNetCalc$ calculated frequency response and the measured frequency response of the XBand microstrip filter match.

For the application of the techniques presented in this paper to plane wave propagation, especially through a Plasma see the paper by the author [4].

For the C Code for the Transmission Line Network Calculation Program

(*TransNetCalc*) **Visit:**

https://www.ccdsp.org/TransTopCalc_Site/index.html Also available on GitHub [2]

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[1] Sasan Ardalan. In *Modeling and Analysis of Transmission Line Networks*. Center for Communication and Signal Processing NC State Umiversity, 1987-2016.

[2] Sasan Ardalan. In *Computer Modeling of Transmission Line Networks*. https://github.com/silicondsp/TransNetCalc, 1987-2024.

[3] Sasan Ardalan. In *Two Stage WiFi 5GHz to 10GHz XBand Communication System*. https://vixra.org/abs/2412.0002, 2024.

[4] Sasan Ardalan. In *Computer Modeling of Plane Wave Propagation Through Dielectric Sections and Plasmas*. https://vixra.org/abs/2412.0028, 2024.

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[6] Robert Grover Brown, Robert A. Sharpe, William Lewis Hughes, and Robert E. Post. In *Lines, Waves, and Antennas, Second Edition*. Ronald Press Company, 1973.

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[8] D.G. Messerschmitt. In *Transmission line modeling program written in C*. IEEE Journal on Selected Areas in Communications,Vol. SAC-2, pp. 148-153, January 1984.

[9] Gerald E. Sobelman and David E. Krekelberg. In *Advanced C Techniques and Applications*. Que Corporation, Indianapolis, Indiana, 1985.

Chapter A

TransNetCalc Modeling of 10 GHz Bandpass Filter

The complete details for modeling the 10 GHz microstrip band pass filter in TransNetCalc is provided in this Chapter.

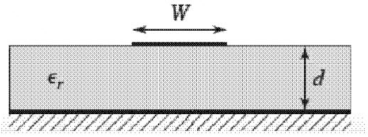

Figure 22: Microstrip

The ASCII File describing the 10 GHz Microstrip Bandpass Filter in Figure 21 is provided below.

```
n1 n2 n3
n3 n4 n5
n5 n6 n7
n7 n8 n9
n9 n10 n11
n11 n12 n13
n13 n14
n2
n4
n6
n8
n10
n12
n14
end
n1 mstrip50 0.004202
n2 mstripZ1_paral_6GHz 0.00406119 0 0
n3 mstrip50 0.0000
n4 mstripZ1_paral_6GHz 0.00406119 0 0
n5 mstrip50 0.004202
n6 mstripZ2_paral_6GHz 0.00414973 0 0
n7 mstrip50 0.0000
n8 mstripZ2_paral_6GHz 0.00414973 0 0
```

```
n9 mstrip50 0.004202
n10 mstripZ3_paral_6GHz 0.00406119 0 0
n11 mstrip50 0.0000
n12 mstripZ3_paral_6GHz 0.00406119 0 0
n13 mstrip50 0.004202
n14 R50_L0.00000 0 open
```

The microstrip transmission line sections are detailed below.

```
mstrip50 0.552507 0.254 3.66 0.01
mstripZ1_paral_6GHz 1.28422 0.254 3.66 0.01
mstripZ2_paral_6GHz 0.763286 0.254 3.66 0.01
mstripZ3_paral_6GHz 1.28422 0.254 3.66 0.01
mstripZ1_paral_4GHz 2.13711 0.254 3.66 0.01
mstripZ2_paral_4GHz 1.33575 0.254 3.66 0.01
mstripZ3_paral_4GHz 2.13711 0.254 3.66 0.01
```

The TCL script for calculating the frequency response is shown below.

```
tload microstrip_bpf_paral_6GHz.top T

cx 1 0 gen
cx 50 0 zs

set freq 4e9

set fBegin 4e9

set fEnd 60e9

set df [expr ($fEnd-$fBegin)/1000.0 ]

for { set i 0 } { $i<1000 } {incr i} {
set f [expr $fBegin +$i*$df ]
tcalc $T $f $gen $zs
nz0gamma $T n7 z0 gamma
nvalues $T n7

cxmult $n7_zl $n7_il vn7
cxgetpolar $vn7 mag ang

ninfo $T n7
set fGHZ [expr $f1e-9] set magdB [expr 20log($mag)/log(10)]
lappend results [list $fGHZ $magdB ]
}

puts “RESULTS:”
foreach value $results {
puts -nonewline [lindex $value 0]
puts -nonewline “\t”
puts [lindex $value 1]

}

puts “Storing results in microstrip_bpf_paral_6GHz.dat”
set out [open microstrip_bpf_paral_6GHz.dat w ]
foreach value $results {

puts -nonewline $out [lindex $value 0]
puts -nonewline $out “\t”
puts $out [lindex $value 1]

}

close $out
```

Chapter B

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